Dunwoody High School

Summer Review

AP Calculus AB

(Solve as many questions as you can and the rest will be discussed in class)

1. This packet is to be handed in to your Calculus teacher on the first day of the semester.
2. All work must be shown on separate paper attached to the packet.
3. Graphs must be drawn on graph sheets.

Things to remember:

Linear forms: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

Quadratic forms: $y = ax^2 + bx + c$ $y = a(x - h)^2 + k$ $y = a(x - p)(x - q)$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin(2x) = 2 \sin x \cos x$

$\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

$\cos(2x) = \cos^2 x - \sin^2 x$

$= 1 - 2 \sin^2 x$

$= 2 \cos^2 x - 1$

Exponential Properties: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

Logarithms: $y = \log_a x$ is equivalent to $a^y = x$

Logarithmic Properties: $\log_b mn = \log_b m + \log_b n$ $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b (m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

For #1- 4, write an equation for each line in point-slope form and sketch the graph.

1. Containing (4, −1) with a slope of $-\frac{2}{3}$.

2. Containing the points (6, -3) and (-3, 2).

3. Write an equation of a line passing through (3, 5) that is parallel to $y = 2x - 3$.

4. Write an equation of a line passing through (4, −2) that is perpendicular to $y = -2x - 5$.

Graph, factor, and solve

5.Graph y = $x^2 - 4x - 12$ and solve $x^2 - 4x - 12 = 0$ by factoring.

6. Graph y = $x^2 + 6x + 9$ and solve $x^2 + 6x + 9 = 0$ by factoring.

7. Graph y = $9x^2 - 4$ and solve $9x^2 - 4 = 0$ by factoring.

8. Graph y = $4x^2 - 12x + 9$ and solve $4x^2 - 12x + 9 = 0$ by factoring.

Find the value of $\frac{f(x+h) - f(x)}{h}$ in the most simplified form.

9. f(x) = 3x + 4

10. f(x) = -2x – 1

11. f(x) = $x^2 + 2x$

12. f(x) = $-3x^2 + 5x - 4$

13. f(x) = $\frac{2}{x+1}$

Absolute value functions

14. Write y = | x | as a piecewise function and graph it.

15. Write y = |x + 2| as a piecewise function and graph it.

16. Write y = |3x - 2| as a piecewise function and graph it.

17. Write y = $\frac{|x-3|}{x-3}$ as a piecewise function and graph it.

Graph the function and write its domain

18. Graph y = $\frac{1}{x}$ and write its domain.

19. Graph y = $\frac{1}{x-2}$ and write its domain.

20. Graph y = $\frac{x^2-1}{x+1}$ and write its domain.

Natural logarithm

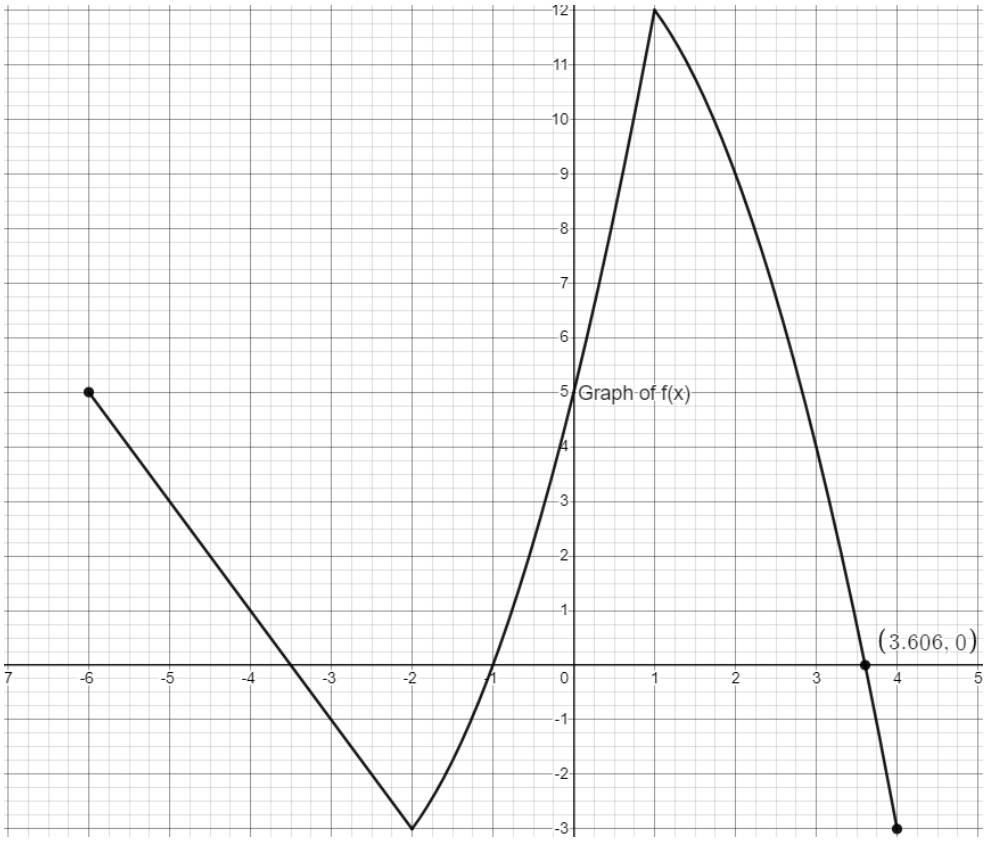
21. Solve for x: ln(3x + 5) = 3.4

22. Solve for x: ln$\left(e^{5x+4.2}\right)$ = 7.26

23. Solve for y: ln(y) = ln(x – 2) + ln c

24. Solve for y: ln(y + 3) = -x + ln c

25. Solve for y: ln(3 – 2y) = tan2x + ln c, given that y(0) = -1.



By using the graph of the function f(x), answer the following questions:

26. For what values of x, f(x) = 0.

27. For what values of x, f(x) > 0.

28. For what values of x, f(x) < 0.

29. For what values of x, f(x) is increasing.

30. For what values of x, f(x) is decreasing.

Trigonometry

Solve without using a calculator.

31. $\sin \pi$ = $\cos\left(\frac{\pi}{2}\right)$ = $\cos 180^0$ =

32. $\tan\left(-\frac{3\pi}{4}\right)$ = $\sin 90^0$ = $\cot 225^0$ =

33. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ =

34. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ = $\cot^{-1}(1)$ =

35. Solve: $\sin x = \cos x$; $x \in [0,\ \pi]$

36. Solve: $\sin 2x = \sin x$; $x \in [0,\ \pi]$

37. Solve: $\sin^2 x - 2\sin x + 1 = 0$; $x \in [0,\ 2\pi)$

Application Problems

38. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner of side x inches and fold up the sides to make an open-topped box.

a. What is the length of the box in x?_____

b. What is the width of the box in x?_____

c. What is the height of the box in x?_____

d. What is the volume of the box in x?_____

39. A square sheet of tin measures 24 inches. Suppose you cut a square out of each corner of side x inches and fold up the sides to make an open-topped box.

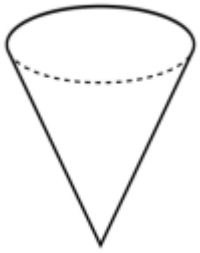
a. What is the length of the box in x?_____

b. What is the width of the box in x?_____

c. What is the height of the box in x?_____

d. What is the volume of the box in x?_____

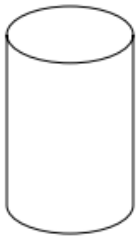
40. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water. Suppose r is the radius and h is the height of the water at the time t. Write an expression for the volume of the water at the time t in terms of its…



a. radius Volume v(r):_____

b. height Volume v(h):_____

41. A cylindrical can that will hold 1000 cubic centimeters. Suppose r is the radius and h is the height of the cylindrical can. Write its lateral surface area and total surface area in terms of r.

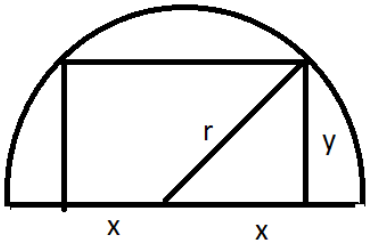


a. Lateral surface area *l*(r) = _____

b. Total surface area *t*(r) = _____

42. A rectangle is inscribed in a semicircle of radius 5 units with the longer side (base) on the diameter. If the height of the rectangle is y units, then write its area as a function of y.

Area of the rectangle a(y) = _____



Simplify

43.Simplify: $\dfrac{\frac{1}{2x}-1}{x}$

44. Simplify: $\dfrac{\frac{1}{x}+\frac{x}{x+1}}{\frac{x}{x+1}}$

45. Simplify: $\dfrac{\frac{x}{2}+\frac{3x-1}{3x}}{\frac{2x}{2x+1}-\frac{x}{3}}$

Solve algebraically and verify your answers by graph.

46. Solve: $y = 3x – 4$ and $y = -2x + 6$

47. Solve: $y = x^2$ and $x = y^2$

48. Solve: $y = 2x^2 – 4x + 3$ and $y = -2x + 7$

49.Solve: $y = -2(x – 3)^2 +1$ and $y = -2x +7$

50.Solve: $y= 2x^2 – 28x + 94$ and $y = -(x – 7)^2 +8$